ISI – Bangalore Center – B Math - Physics II – End Semestral Exam
Date: 31st December 2021. Duration of Exam: 3 hours
Total marks: 60

1. **(a)** The internal energy E(T) of a system at a fixed volume is found to depend on the temperature T as $E(T) = a\,T^2 + b\,T^4$. Find the entropy $S(T)$, as a function of temperature.
   (b) Calculate the temperature after adiabatic compression of a gas to 10.0 atmospheres pressure from initial conditions of 1 atmosphere and 300K (i) for air ($\gamma = 1.4$), (ii) for helium ($\gamma = 5/3$) (assume the gases are ideal).
   (c) One mole of a van der Waals' gas undergoes a reversible isothermal expansion from a volume v_i to a volume v_f. Calculate the amount of heat transferred in this process.
   (d) Calculate the Joule coefficient for a van der Waals' gas. Describe the process of Joule Thompson expansion for a gas (not necessarily an ideal gas) and show that enthalpy is conserved in this process.
   (e) Describe the four thermodynamic potentials and generate the Maxwell relations.

   Or

   10 litres of gas at atmospheric pressure is compressed isothermally to a volume of 1 litre and then allowed to expand adiabatically to 10 litres.
   (a) Sketch the processes on a pV diagram for a monatomic gas.
   (b) Make a similar sketch for a diatomic gas.
   (c) Is a net work done on or by the system?
   (d) Is it greater or less for the diatomic gas?

   (e) Describe the four thermodynamic potentials and generate the Maxwell relations.

   2+2+2+2+4=12

2. **(a)** What do you mean by thermodynamic probability? How the entropy is related with thermodynamic probability?
   (b) Find the expression for the Carnot efficiency for a perfectly reversible Carnot cycle using an ideal gas.
   (c) Which of the following is an exact differential? $(i) dU(x,y) = 2xdx + 3xydy$.
   (ii) $dU(x,y) = -\frac{1}{y}dx + \frac{x}{y^2}dy$.

   Consider a model of rubber band as a chain of N links of negligible width, with each of the links being of length a and each link capable of pointing either to the left or to the right. if the rubber band is thermodynamically described by its length L under tension f at temperature T,

(d) Show that the number of states is given by $\Omega(N, L, a) = N! \frac{N!}{\left(\frac{N}{2}-\frac{L}{2a}\right)!\left(\frac{N}{2}+\frac{L}{2a}\right)!}$.

(e) Find the entropy in the limit of large N, as a function of N, L and a.

2+2+2+3+3=12

3. Explain microstates and macrostates. What is the phase-space dimension of N particles in three-dimensions?

Consider an isolated system of four non-interacting spins labelled 1,2,3 and 4. With magnetic moment μ, interacting with an external magnetic field H. Each spin can be parallel ("up") or antiparallel ("down") to H, with the energy of a spin parallel to H equal to $\epsilon = -\mu H$ and the energy of a spin antiparallel to H equal to $\epsilon = +\mu H$. Let the total energy of the system be $E = -2\,\mu H$.

(a) How many microstates of the system correspond to this macrostate? Enumerate these microstates. What is the probability that the system is in a given microstate in equilibrium?

(b) What is the probability that a given spin points up? Use this probability to compute the mean magnetic moment of a given spin in equilibrium.

(c) What is the probability that if spin 1 is `up', spin 2 is also `up'?

(d) A system of four weakly interacting distinct particles is such that each particle can be in one of four states with energies ϵ, $2\,\epsilon$, $3\,\epsilon$ and $4\,\epsilon$ respectively. If the system has total energy $12\,\epsilon$, what is the entropy of the system? For what possible values of total energy is the entropy of the system zero?

1+3+3+2+3=12

4. **(a)**Consider a Maxwellian Distribution of the velocity of the molecules of an ideal gas. Let v_{mp}, v_{avg} and v_{rms} denote the most probable velocity, mean velocity and the root mean square velocity respectively. Find the relation between v_{rms}, v_{mp}, and v_{avg}. Show them (approximately) in the distribution curve.

(b) Calculate the rms speed of a nitrogen molecule at room temperature. [One mole of N_2 has a mass of 28g. The Boltzmann constant is 1.38×10^{-23} and $T = 300K$**.**

(c) Find the probability for a molecule to move with at least twice the average velocity. Help: $\int_{2.26}^{\infty} x^2 \exp(-x^2)\, dx \approx 0.0075$

(d) Derive the ideal monatomic gas law using kinetic theory. You may use the equipartition result $U = \frac{3}{2} N k_B T$.

4+2+2+4=12

5. **(a)** Two simple harmonic motions at right angles are represented by $y = 2\sin 2\pi t$ and $z = 4\sin(2\pi t - \frac{3\pi}{4})$. Find the equation for the resultant path and plot this path. Explain the figure in detail. The phase velocity of waves in a certain medium is represented by $v = a + C_1\lambda$ where a and C_1 are constants. What is the value of group velocity?

(b) What is the relation between phase difference and path difference? What is the path difference for destructive interference?

(c) What is the effect on the interference fringes in a Young's double-slit experiment due to each of the following operations: (i) the screen is moved away from the plane of the slits; (ii) the (monochromatic) source is replaced by another (monochromatic) source of shorter wavelength.

(d) In a Young's double-slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm. Determine the wavelength of light used in the experiment.

Or

A thin film has a thickness of 0.04650 cm and a refractive index of 1.5230. Find the angle ϕ at which the dark fringe 122.5 will be observed if monochromatic light 6560 A^o is used as an extended source.

(e) Light (600 nm) strikes a grating ruled with 300 lines/mm. What is the angular deviation of the 2^{nd} order bright fringe?

4+2+2+2+2=12